THE EFFECT OF THE CALCULATOR ON STUDENT ACHIEVEMENT IN GRAPHING LINEAR FUNCTIONS

By

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The purpose of this study was to determine the effectiveness of the graphing calculator on the acquisition of skill in graphing linear functions. It also addressed the impact of the graphing calculator on homework assignment, on the development of spatial visualization ability, and on the development of attitudes toward mathematics or toward calculator use in mathematics instruction. Gender differences in these areas were also considered.

Fifty-seven students from two intact classes of eighth grade prealgebra participated in the study. Data were collected by means of a pretest and a posttest about linear functions, the Card Rotation Test, homework assignments, and two questionnaires about attitudes.

General Linear Model procedures were applied to the data to test the effectiveness of the graphing calculator on students' development of graphing skills in linear functions, on the development of spatial visualization ability, and on the development of attitudes toward mathematics or toward calculators. Data related to homework assignments were analyzed using analysis of variance.

Results of these analyses revealed the following:

1. There were no significant differences between treatment groups or gender groups in mathematics achievement in linear functions.

2. Students using the graphing calculator did homework assignments more efficiently and in less time than those not using the graphing calculator.

3. Students receiving the instructional treatment by means of the graphing calculator exhibited significant gains in spatial visualization skills and in attitudes toward calculators. No significant differences were found between gender.

4. There were no significant differences between treatment groups or gender groups in attitudes toward mathematics.

Based on these findings it may be concluded that it appears that the graphing calculator can be an effective tool that should be incorporated into the mathematics curriculum. However, further investigations are needed, including longer term studies, studies using larger samples, and studies in other areas of mathematics.

CHAPTER 1

INTRODUCTION

> We are on the threshold of an exciting and revolutionary period, in which the scientific study of man will be put to work in man's best interest. Education must play its part. It must accept the fact that a sweeping revision of educational practices is possible and inevitable.
>
> (Skinner, 1954, p. 89)

Mathematics teachers, mathematicians and mathematics educators have been searching for ways to improve instruction to ensure students' understanding of mathematical concepts. One way to carry out this function is by using technology, in particular computers and calculators, in the mathematics classroom. In fact, it has been recommended that the calculator should be used in all areas of instruction, including testing (Heid, 1988; NCTM, 1989).

When properly used, technology has the potential to encourage students to be more alert, curious, creative, investigative, and interested. With the aid of technology students can discover principles and make generalizations. Technology can help to stimulate thought, develop students' minds and make them more prepared to live more functionally in this technological age.

During the past years several educators and organizations have published articles and books that make a strong case for the use of technology in the mathematics classrooms. It has been said that because we are living in a technological era we must redefine "basic education." Teaching mathematics should no longer merely focus on the ability to add, subtract, multiply and divide as it was in the past. Instead, educators should focus on concepts and skills useful in a highly technological world and should use technology to assist in educating our students for the world of the future. Educators should adopt new methods and curricula using technology to meet the present-day needs of high-level mathematics (Bitter, 1987).

Recently computers have become affordable and even more recently the graphing calculator has put the power of what was only possible with a desktop microcomputer in the hands of the students by allowing each student to have graphing capability for classwork as well as homework assignments.

Little research has been done using this technology. There is a need to establish the effectiveness of the use of the hand-held graphing calculator that has the power of a computer. This study is designed to begin to provide data with which mathematics educators and teachers can make decisions about the use of the graphing calculator to teach concepts related to linear functions. While the concept of the graph of a function and the associated relationships are important, the main focus of this study is the method of presentation of the concept.

Purpose

The study presented a comparison of treatments between two groups of students. The purpose of the study was to test the effect of using the graphing calculator as a tool in instruction on four areas (a) learning to graph linear functions, (b) homework assignments, (c) spatial visualization ability, (d) students' changes in attitudes toward mathematics and toward using technology to learn mathematics, and (e) sex differences.

Outline of the Study

The central questions of the study dealt with the effects of the graphing calculator on the students' acquisition of graphing skills in linear functions. It also focused on the effectiveness of the calculator as a tool for use in homework assignments, on students' acquisition of spatial visualization ability and students' changes in attitudes toward calculators or toward mathematics.

The subjects in this study were students from two intact classes of eighth grade prealgebra from Lincoln Middle School in Gainesville, Florida. Students were tested at the beginning of the study on the five areas described above. After an instructional treatment consisting of a unit of eight lessons related to linear functions, students were tested again on the same skills to establish the effects of the experimental treatment.

Two statistical techniques were used to analyze the data collected during the study. General Linear Model (GLM) procedures were used to test the effect of the graphing calculator on developing graphing skills in linear functions, on developing spatial visualization ability, and on developing attitude changes toward mathematics or toward calculator use in learning mathematics. Analysis of variance was used to analyze the data related to homework assignments.

Research Questions

The purpose of the study was to answer the following research questions:

1. Is there a difference in students' abilities to graph linear functions if the graphing calculator is used in instruction and students are allowed to use the calculator in all phases of learning including homework

assignments?

2. Does using a graphing calculator when learning linear functions have an effect on students' spatial visualization ability?

3. Does using the graphing calculator in the classroom have an effect on a student's attitude toward mathematics and/or toward using a calculator?

4. Is there a difference in the performance of males and females in the four areas described earlier in the chapter when the graphing calculator is used in instruction?

Research Hypotheses

The above questions generated the following hypotheses:

H_1: There is no significant difference in performance between the control and the experimental group in graphing linear functions and in solving problems related to linear functions.

H_2: There is no significant difference in performance between males and females in graphing linear functions and in solving problems related to linear functions.

H_3: There is no significant difference in spatial visualization ability between the control and the experimental groups.

H_4: There is no significant difference in spatial visualization ability between males and females.

H_5: There is no significant difference in attitude changes toward mathematics between the control and the experimental groups.

H_6: There is no significant difference between males and females in attitude changes toward mathematics.

H_7: There is no significant difference between the control and the experimental groups in attitude changes toward calculators.

H_8: There is no significant difference between males and females in attitude changes toward calculators.

H_9: There is no significant difference in the number of homework problems completed between the control and the experimental groups.

H_{10}: There is no significant difference in the percent of homework problems solved correctly between the experimental and control groups.

Significance of the Study

The National Council of Teachers of Mathematics (NCTM, 1980) and the National Advisory Committee on Mathematical Education (NACOME, 1975) have recognized the potential contribution of the calculator as a valuable instructional aid. Both have also recommended that mathematics programs should take full advantage of the power of calculators at all grade levels. The present study examined educational aspects of a new technology, the graphing calculator with the power of a computer. The intention of the study was to demonstrate that the graphing calculator is an effective instrument to assist and enhance mathematics instruction. It also wanted to demonstrate a time saving strategy for teaching a traditional topic in the prealgebra curriculum.

Limitations

Several limitations bounded the present study. One is that intact classes were used. As a consequence generalization of the results must be made with caution. A second limitation was the sample size, which was limited to only two groups of eighth grade students. A third limitation was the time allowed to the study. This could have the possibility of producing novelty or Hawthorne effects.

Summary

A rationale for the study and a brief description of the research procedures were presented in Chapter 1. The literature that deals with (a) technology in mathematics instruction, (b) spatial visualization ability, (c) the teaching of graphing skills, and (d) attitudes will be reviewed in Chapter 2. The description of the research design is presented in Chapter 3. The data collected during the study are analyzed in Chapter 4. Conclusions, a discussion of the findings, and some recommendations for further research are presented in Chapter 5.

CHAPTER 2

REVIEW OF LITERATURE

The literature search for this study is divided into four sections:

1. Technology in mathematics instruction
2. Spatial visualization ability
3. The teaching of graphing skills
4. Attitudes

Use of Technology in the Teaching of Mathematics

Educators agree that a critical problem that mathematics education is facing at the present time is the need for curricula and instructional methods that respond to the influence of technology on the teaching and learning of mathematics (Corbitt, 1985). New methods and curricula are necessary to meet today's needs of both the high level of mathematics and the high level of thinking skills demanded by business and industry in this technological age (Bitter, 1987). This opinion is shared by Hoffer (1987), who believes that in these days of rapid technological advancement it is essential to offer students more effective learning experiences and to consider ways in which teachers can best utilize technology in schools. He suggests the use of calculators and computers which can free students from the shackles of tedious computation and can focus their learning on concept and skill development. For Noss (1988) technology can be the instrument for bridging the gap between what mathematics students do as part of their everyday experience and the mathematics that they learn at school.

In 1984 the National Science Foundation (NSF) sponsored a conference at NCTM headquarters to consider "The Impact of Computing on School Mathematics." Based on the results of that conference it was suggested that the major influence of technology in mathematics education is its potential to shift the focus of instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures, and problem solving skills. The steering committee of the conference described above also concluded that

> applications of calculators, computers and other electronic information technology are reshaping the fundamental methods of doing and teaching mathematics. When used as tools for arithmetic and for the analysis of graphic or symbolic data, calculators and computers offer powerful new approaches to familiar problems and access to entirely new branches of mathematics. (Corbitt, 1985, pp. 243-244)

In addition, they make a strong case for the use of the graphing capabilities of the computer in the mathematics classroom. They recommended that

> Middle school mathematics program should take advantage of the visual display capabilities of computer graphics that support and underscore the importance of informal geometry objectives. Transformations, mensuration formulas, and visualization can be vividly illustrated by using computer graphics. (Corbitt, 1985, p. 246)

In fact, since 1981 several investigations have shown that computer graphics can be used interactively in the classroom to enhance the teaching of mathematics. Fransen (1981) showed how useful the microcomputer is in enhancing trigonometry instruction in secondary school by graphical solutions of triangles and graphical displays of functions. Rudnytsky (1982) concluded that computer graphics could be used to teach concepts in mathematics more easily by manipulating relationships among visual objects. Rieber (1983) used second grade students to investigate the effects of Logo's turtle graphics on providing systematic thought and on teaching simple geometric concepts. He found that the group using the computer performed significantly better than the traditional group in thinking skills in geometry.

Another piece of technology that has been highly recommended to enhance mathematics instruction is the calculator. It has been suggested that calculators should routinely be available to students in all activities associated with mathematics learning, including testing (Corbitt, 1985; Heid, 1988; NACOME, 1975; NCTM, 1980, 1989). For Williams (1987) probably the only true obstacles to the use of calculators in the teaching of mathematics are our attitudes and beliefs.

Reys and Reys (1987) give some suggestions for teachers when using calculators in the mathematics classroom. They recommend that teachers should become knowledgeable about the research on calculators, recognize the intangible benefits of calculator use, develop an "open" calculator policy, allow students to explore with calculators and never require students to use calculators to check paper-and-pencil computation. They emphasize that although checking is perhaps the most common use of calculators, it is counterproductive; since checking paper-and-pencil work does not reflect the real-world use of a calculator.

Others have observed that it is quite ironic that as the calculator availability is reducing the need for computation for most people in everyday life, schools are increasing their emphasis on paper-and-pencil computational proficiency (Wiebe, 1987; Williams, 1987). This does not mean that computational skills are unimportant. The acquisition of these skills solely via paper-and-pencil methods is what can and should be changed (Wiebe, 1987; Williams, 1987).

Similar observations and recommendations were made several years ago. As early as 1977 Bell, Esty, Payne, and Suydam analyzed the most frequently cited reasons for using calculators in the school. These reasons included

1. They aid in computation;
2. They facilitate understanding and concept development;
3. They lessen the need for memorization;
4. They help in problem solving since problems can be more realistic and the scope of problem solving can be enlarged;
5. They motivate learning by encouraging discovery, exploration and creativity; and perhaps the most compelling reason;
6. They exist. Calculators are here to stay in the real world; we cannot ignore them.

Despite all of those observations and recommendations there is still much opposition to the use of calculators in the mathematics classroom. Some believe that such use will harm mathematics achievement. In the study cited above (Bell et al., 1977) several reasons against the use of calculators were analyzed. Among the reasons against the use of calculators in instruction is that the emphasis in instruction is often on the product rather than the process and that calculators could be used as substitutes for developing computational skill.

However, research has shown that this is not the case. As far back as 1937 Betts concluded that students can successfully operate calculating machines in the elementary grades with no loss of computational skills (Shult, 1981). More recently Hembree and Dessart (1986) analyzed the results of 79 studies comparing achievement and attitudes of students using calculators against those not using calculators in grades K-12. They found that students using calculators in their mathematics classrooms as part of their instruction, maintained their skills with pencil and paper without apparent harm. They also found that students using calculators had better attitudes toward mathematics. Only in grade 4 did sustained calculator use appear to hinder the development of basic skills. However, Dean (1981) studied the effect of using calculators as an instructional aid in teaching the basic multiplication facts to fourth graders and found no significant difference in achievement or retention between groups using calculators for all computation or only for checking and those not using calculators at all.

Several other studies have investigated the effects of using calculators on the problem-solving process. Szetela and Super (1988) working with seventh grade students found that students using calculators in the problem-solving process scored significantly higher on attitude toward problem solving and on paper-and-pencil computation. Murphy (1981) studied how calculator use would affect achievement and attitude toward problem solving in seventh-grade mathematics. She found that students with unrestricted use of calculators achieved higher problem-solving scores than students not using calculators for instruction or tests. Hedren (1985) found that classes in Sweden which used calculators whenever they could be of use were as competent as control classes in mental arithmetic and calculations with simple algorithms and had better understanding of numbers and problem-solving strategies. Other studies have found that students using calculators while being instructed in problem solving produce significant improvement in less time than students who relied solely on paper-and-pencil computation and that the calculator enhanced the implementation of strategies to increase understanding of problems and problem-solving procedures (Halasz, 1984; Kelly, 1985; Leechford & Rice, 1984; Shult, 1980; Stewart, 1981).

The effect of using calculators in classes which involve students with some type of disability has also been investigated. The results of those studies reveal that students using calculators outperformed those not using calculators, in both mathematics achievement and attitudes. In addition, it was also found that calculator use reduced disruptive behavior (Advani, 1972; Golden, 1983; Horton, 1985).

Spatial Visualization Ability

The literature related to the spatial visualization ability is extensive. This makes the search quite difficult. Another difficulty is the fact that there has not been an agreement on what factors are involved in spatial-visual ability. In his earliest studies Thurstone (1938) believed that there was only one spatial factor. According to him this factor was the facility with spatial and visual imagery. Twelve years later he convinced himself that spatial visualization was composed of two factors which were independent one of the other. He named those factors as spatial relations and visualization (Thurstone, 1950).

Other investigators have also concluded that there are two factors involved in spatial visualization. However, they have labeled them differently from Thurstone

(French, 1951; Guilford, Fruchter, & Zimmerman, 1952). For them spatial-visual ability is composed of the factors, spatial orientation and spatial visualization.

In 1957 Michael, Guilford, Fruchter, and Zimmerman identified an additional component of the spatial-visual aptitude. The three factors and their definitions are as follows:

1. Spatial relations and orientations denote the ability to understand the relationships between elements in a given stimulus pattern with respect to the body of the observer.

2. Visualization is the ability to mentally manipulate visual objects involving specific movements.

3. Kinesthetic imagery is the ability to discriminate left and right with respect to the location of the body of the individual. Smith (1964) concluded that spatial ability is not composed of a unique factor but by a combination of two or more factors. The number of those factors and the name given to them depend on the authors.

The visualization factor is the one of interest in the present study. In it an individual needs to develop the ability to mentally rotate, turn, or flip one or more objects. The Card Rotation Test which has been used in other investigations to measure this ability was also used in the present study to determine the effectiveness of the graphing calculator on the development of spatial visualization ability. A detailed description of this test is included in Chapter 3.

The relationship between spatial-visual ability and mathematical learning has been of special interest to many mathematics educators. Although this relationship seems to be logically evident, data showing a positive relation have been less convincing. Guilford, Green, and Christensen (1951) found that spatial visualization ability helped in solving problems in mathematics. For French (1951, 1955) the success in mathematics achievement depends on the use of spatial visualization skills. Krutetskii (1971, 1976) identified several problems that require spatial visualization to solve them. His research measured the extent to which a student uses more or less visual ideas in solving a mathematics problem. He concluded that those students good in problem solving usually used spatial ideas to solve the problems.

A positive relationship between mathematics achievement and spatial visualization has been discovered (Aiken, 1973; Guay & McDaniel, 1977; Shieh, 1985). The same has been proved in relation to the relationship between geometry ability and spatial visualization ability (Battista, 1981; Battista, Wheatley, & Talsma, 1982).

However, other educators feel that data show a negative relationship between those abilities. Very (1967) found that research on spatial ability has failed to show a significant correlation of spatial visualization with mathematics performance. Werdelin (1958, 1961) failed to find a positive correlation between spatial ability and computation and between spatial ability and geometry achievement. Lim (1962) obtained the same results with relation to spatial ability and geometry. Lean and Clemens (1981) found that spatial ability and knowledge of spatial conventions had little effect on scores for pure and applied mathematics tests.

Other investigators have studied the effects of spatial visualization instruction on student performance. Tillotson (1985) working with sixth-grade students found that instruction in spatial visualization ability was effective in developing problem-solving skills. In a study by Smith and Schroder (1979) it was found that fourth graders, regardless of sex, who had received instruction in spatial visualization outperformed those who had received no instruction. Similar results were obtained by Gabel and Enochs (1987) in the presentation of the volume concept and by Small and Mort (1983) in an organic chemistry course. In contrast, Frienlander (1984) found that spatial visualization instruction did not have a significant effect on achievement in similarity tasks.

The effects of technology, in particular the use of computers in the acquisition of spatial visualization ability has also been of interest to many educators. Kantowski (1985) analyzed various computer programs used to teach transformational geometry. She concluded that this transformational approach makes geometry a dynamic subject which can help develop the students' spatial visualization ability. Kizer (1987) working with college students studied the effect of the computer in the teaching of graphs of linear inequalities. He found that students using computer-enhanced instruction had significantly higher achievement and more positive attitudes than those not using the computer. In addition, he concluded that the computer was especially effective for those with high spatial ability. Similar results were obtained using Logo programming with fifth and sixth graders (Miller, Kelly, & Kelly, 1988). Chien (1986) investigated the effectiveness of dynamic and interactive microcomputer graphics on children's acquisition of spatial visualization ability. In particular he was interested in the mental rotation in two dimensions. The results of the study showed that using graphic spatial programs on the microcomputers was slightly more effective than hands-on manipulative activity.

The difference in performance between males and females in spatial ability tasks has also been investigated. Several studies about this matter have shown that males tend to perform better than girls in spatial visualization problems (Bolk & Kolskowski, 1973; Fennema & Tartre, 1986; Guay & McDaniel, 1977; Miller, 1967; Pepin, Beaulieu, Matte, & Leroux, 1988). However, other studies have shown that this is not generally true. In 1972 Maccoby and Jacklin found that sex differences in spatial ability were very small and that it was not until the ages of 10 or 11 when males began to show some superiority over girls. Those findings contradicted previous studies by Maccoby in which she concluded that male superiority began at early ages and continued until the college years (Maccoby, 1966). Some years later she had changed her conclusions. She then believed that the male superiority was only in a single factor of spatial-visual ability (visualization) and that it was noticeable only during the adolescence (Maccoby & Jacklin, 1974). For Werdelin (1961) girls are better proving verbal theorems but are less able to translate words into figural images and use them to solve a problem. In 1977 Fennema and Sherman found no significant difference in spatial visualization tests between males and females. This was true when boys and girls were carefully matched according to mathematics and related courses they had studied.

Others have studied the relationship between spatial visualization ability and mathematical achievement across various ethnic groups. In 1970 Kabanova-Meller found no significant difference on spatial ability between Russian boys and girls. This was true in grades 4, 5, and 6. Berry (1966) and Kleinfeld (1973) studied the spatial-visual ability of Eskimo students. Both concluded that even though it appeared that Eskimo students had a well-developed spatial ability, there was no significant difference between males and females. More recently Shieh (1986) investigated if spatial visualization varies among Chinese-American, Hispanic-American, and Caucasian seventh- and eighth-grade students. Among the findings are the following:

1. Chinese-American males scored at a significantly higher level than did Chinese-American females.

2. There was no significant sex difference in Caucasian and Hispanic groups.

3. Spatial visualization was a significant predictor of mathematics achievement for the student population of the study. Iben (1986) compared the performance in spatial visualization ability among U.S. students and

Japanese students. It was found that the Japanese mean scores for space relations were higher than those of the U.S. counterparts. It was then concluded that the U.S. classroom experiences do not appear to develop space relationships as fully as in Japan. Similar observations are made by several other educators. In his book Smith (1964) concluded that spatial abilities make an important contribution to mathematics ability that is usually ignored in public school education. Others claim that in the American classroom the emphasis is on the verbal-logical aspect of mathematics and mathematical representations which either encourage or require the use of spatial visualization skills are not used (Fennema, 1981; McKim, 1972; Shear, 1985).

The Teaching of Graphing Skills

> Graphs . . . have assumed an increasingly important role in our society. They present concepts in a concise manner or give at a glance information which would require a great deal of descriptive writing. They often distill a wealth of information into a small amount of space. (Weintraub, 1967, p. 345)

Learning how to graph and how to interpret graphical information has become a necessity for every individual. It is essential to many subject areas and it is a basic tool for scientists, engineers, and other professionals.

However, many middle and secondary school students have not developed an understanding of the relationship between a mathematical function and its graph. Data from the recent Second International Mathematics Study suggest that American high school students have not mastered the basic precalculus topics of function, graphing, and problem solving (Waits & Demana, 1988).

Early research on graphing focused its attention on how students read different types of graphs. It was found that at all grade levels circle and bar graphs were easier to read than line graphs (straight lines and other types of curves). This is perhaps due to the fact that line graphs are used to represent relationships between two variables and their interpretation requires more abstract reasoning ability (Padilla, McKenzie, & Shaw, 1986).

Padilla, McKenzie, and Shaw (1986) examined the ability of students to graph linear functions in grades seven to twelve. They found the seventh- and eighth-grade students scored lower on the graphing measure than students in any other grade. Also, that subjects achieved well on items related to plotting points but not well on items related to interpreting graphs. Based on these results they recommended that training beyond the plotting of points is necessary and that this training should be introduced early in the study of functions. They also concluded that teachers should go beyond the plotting of points when teaching graphing skills, especially if students are going to use graphs for interpreting information.

In a comparison between three different methods (graph, discovery, computation) for improving students' conception of average speed, it was concluded that better instructional procedures are required to assist students in interpreting graphs (Reed & Saavedra, 1986).

Ponte (1984) investigated patterns of achievement and strategies and difficulties of high school students and preservice teachers in functional reasoning, and in reading, constructing and interpreting Cartesian graphs. He found that all groups were able to use graphs to gather numerical information but not to interpret the underlying relationship. He then suggested that the interpretation of complex graphs should be part of the secondary school mathematics curriculum and that students should be encouraged to go beyond point-by-point strategies.

Observing those studies one can realize that the focus on the teaching of graphing skills in the school has been on constructing graphs usually by the time-consuming method of plotting points. There is very little emphasis on the interpretation of graphs. Procedures for teaching for understanding rather than for the development of rote manipulative techniques are needed.

The graphing capabilities of the computer and more recently those of the graphing calculator could be very helpful in the development of skills about interpreting graphs. It has been said that the computer and the graphing calculator are two useful tools in producing quick and accurate graphs, eliminating those tasks which are tedious and time-consuming (Waits & Demana, 1988). In addition, it has been suggested that the teaching of graphing via computer could help students to explore and learn mathematics by providing pictorial representations of functions and equations (Bitter, 1987; Kantowski, 1981). For McConnell (1988) computer and calculator graphing tools should elevate graphing to a primary position in the algebra curriculum. He suggested that a computer or a calculator that graphs functions should enable students to have a better understanding of graphs. The NCTM Commission on Standards for School Mathematics (1989) recommends that teachers should reduce attention on paper-and-pencil graphing of equations by plotting points and increase attention on computer graphing to develop conceptual learning.

Research in this area has shown that in fact the computer and the graphing calculator are effective tools in developing graphing skills. Porter (1979) found that computer-generated graphics displays were useful in teaching mathematical concepts to undergraduate students. Phillips (1982) looked at students' interpretation of graphs presented by computer and concluded that the microcomputer can be an effective teaching aid in the area of graphing interpretation. Marty (1986) used 425 students to study the effect of a computer game on graphing ability in secondary school algebra. Some students used the Algebra Arcade game while others used traditional classroom methods. Results showed significant difference in graphing ability favoring the group using the computer game. In the only study conducted using the graphing calculator that this investigator has knowledge of, Browning (1989) developed an assessment instrument to determine and characterize levels of graphical understanding. Over 200 precalculus students from four school districts participated in the study. Some students used graphing technology (graphing calculators or computers) in the classroom while others did not have this technology available. Some of the findings are the following:

1. The group using graphing technology had significantly higher means in the posttest than the group without the graphing technology.

2. Students had great difficulty in communicating their ideas and problem-solving strategies.

Attitudes

> An attitude can be defined as an enduring organization of motivational, emotional, perceptual and cognitive processes with respect to some aspect of the individual's world. (Kretch & Crutchfield, 1948, p. 152)

Two definitions of attitude used by mathematics educators who have dedicated a great deal of time researching mathematics attitude seem not to differ from the definition described above by two social psychologists. In 1969, Romberg and Wilson described attitude as follows:

> If an individual has a set of predispositions toward an object in the environment (e.g. mathematics, self, school, teacher, etc.), it is reasonable to expect that such predispositions would interact with the perception of the object in such a way as to affect the individual's response to that object. (p. 151)

On the other hand, Aikens (1972) noted the following as he reviewed research about attitudes toward mathematics at that time.

> The term attitude as used in the studies referred to here means approximately the same thing as enjoyment, interest, and to some extent level of anxiety. (p. 229)

In attitudes toward mathematics, one of the aspects of the individual's world in the definition used by Kretch and Crutchfield might be "mathematics in general," "working with fractions," or "geometry." However, it is very difficult to give a definition of attitude toward mathematics that satisfies everyone and therefore each investigator should clarify very well what he or she wants to measure.

Research on mathematics attitude has been very popular among mathematics educators. Among the factors that have been investigated are (a) the relationships between achievement and attitudes toward mathematics, (b) sex differences in attitudes toward mathematics, (c) teachers' influences on students' attitudes toward mathematics, and (d) the effect of technology on developing attitude toward mathematics. Two of the more extensive reviews about research in mathematics attitude made by Aikens (1972, 1976) revealed the following:

1. Although the correlation between attitude and achievement is low, this correlation is positively significant. This was true at all levels of the school system, with students of other countries, and with minority groups in the United States.

2. The correlation between attitude and achievement varies with the grade level and with the sex of the student.
3. Studies related to how teacher's attitude influences student's attitude have been very contradictory.
4. The use of technology to improve attitudes toward mathematics has been somewhat effective.

This last aspect of how technology affects the development of attitudes toward mathematics has been recently the focus of several investigations. In an analysis of 79 studies about the use of the calculator in the mathematics classroom, Hembree and Dessart (1986) found that students using calculators had better attitudes toward mathematics than those not using calculators. Similar results have been obtained more recently in studies in which the computer has been used as the method of instruction (Cook, 1988; Edgar, 1988; Perkins, 1988).

Summary

A review of the literature related to the areas of concern of the present study was presented in Chapter 2. Based on the readings, it may be concluded that since technology is constantly changing, further studies to determine its place in the mathematics curriculum are needed. The present study is a response to this call.

CHAPTER 3

RESEARCH DESIGN

This chapter is divided into three sections. The first section is dedicated to the description of the sample. The second section describes the research instruments used in the study. The third section is a description of the procedures used in the study.

The experimental design used in the study was a Non-randomized Control Group, Pretest/Posttest Design (Ary, Jacobs, & Razavieh, 1985).

Sample

This study was conducted in Lincoln Middle School in Gainesville, Florida. Lincoln Middle School is one of four public middle schools in the city of Gainesville and the only one with a bilingual program among its offerings. For that reason its student population consists of students from all around the world. The sample included in this study was a representation of that diversity of the student population.

Two groups of average eighth-grade students participated in the study. These students were enrolled in two classes of prealgebra. Students from those two classes could not be randomly assigned to control and experimental groups. Intact classes were used. However, random procedures were used to determine the experimental group and the control group.

Research Instruments

The research instruments to be considered are: the pretest, the posttest, the Card Rotation Test to measure spatial visualization ability, and two attitudes scales, one to determine changes in attitudes toward mathematics and one to determine changes in attitudes toward using calculators in mathematics instruction. The instructional lessons constitute the treatment.

Pretest and Posttest

The pretest and the posttest were developed by the investigator for this study. They were reviewed by mathematics educators who were familiar with the purposes of the study, the sample and the instructional lessons to be taught. The pretest (Appendix A) contained 24 multiple-choice exercises. The exercises and the concepts tested on them are the following:

Exercises	Concept
1-5	Slope of a line
6-10	x- and y-intercepts
10-14	Parallelism and perpendicularity
15-16	Comparison between slopes
17-18	Quadrants for a given line
19-20	Use of different scales
21-24	Reflection and rotation

The posttest (Appendix B) was an alternate form of the pretest. Only the order of the exercises and the values used in the problems were changed.

Card Rotation Test

The Card Rotation Test was used to measure spatial visualization ability. Originally designed by Thurstone (1938), this test was included in the Kit of Reference Test for Cognitive Factors published by the Educational Testing Service. The form used in this study was the modified NLSMA version of the original test used in testing students in the School Mathematics Study Group (SMS$) program.

This test measures the students' ability to recognize the relationships among parts of a figure in order to identify the figure when its orientation is changed. The test contains fourteen problems. Each one consists of a given irregular figure followed by eight other representations of the figure. These representations have been either rotated or reflected. The students had four minutes to mark the 112 figures with a "+" if the figure had been rotated or a "-" if the figure had been reflected. An example is given in Figure 3.1.

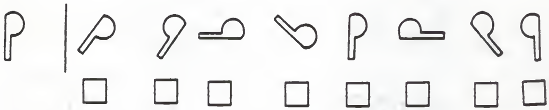


Figure 1

Example of an Exercise of the Card Rotation Test

Attitudes Scales

Two questionnaires (Appendix C) with attitudes scales were used in the study. The purpose of those scales was to determine any change in attitudes toward mathematics and/or calculators due to the experiment. Each one of the questionnaires consisted of 10 statements. The students had to express on a five-point Likert scale the extent of their agreement with their own personal feelings. The five-point scale ranged from 1=strongly disagree to

5=strongly agree. An additional alternative; 9=no response was given for those students who did not want to express their degree of agreement to any or all of the statements.

Kuder-Richardson (K-R 20) formula and Cronbach's coefficient alpha formula were used to determine the reliability coefficient (r) of the instruments in this study. K-R 20 was used for the instruments measuring linear function concepts and spatial visualization ability. Cronbach's formula was used for the attitudes scales. Table 3.1 summarizes the results.

Table 3.1

Reliability of the Instruments

	Pre (r)	Post (r)
Linear function	.58	.60
Spatial visualization	.81	.83
Attitudes (Math.)	.83	.82
Attitudes (Calculator)	.78	.78

Instructional Lessons

A unit consisting of eight lessons (Appendix D) was developed by the investigator for this study. These lessons were adapted from lessons of the book Using the Computer to Teach Algebra (1986). This book was the result of a project developed by the Center of Excellence (Mathematics, Science, Computers and Technology) at the University of Florida. Teachers from Alachua and Marion Counties participated in this project directed by Dr. Mary Grace Kantowski. The lessons were reviewed by mathematics educators familiarized with the study.

The same lessons were used by both the experimental group and the control group. The method of presentation of those lessons was different. For the experimental group the lessons were taught using the graphing calculator, while the traditional method of plotting points using paper and pencil was used to teach the lessons to the control group.

A brief description of each lesson follows:

Lesson 1. Graphing horizontal lines. Students worked with equations of the form y=b, where b is a constant. The concept of y-intercept was introduced.

Lesson 2. Graphing lines through the origin. Equations of the form y=mx and the concept of slope were studied.

Lesson 3. Graphing equations of the form y=mx+b. The focus of this lesson was to let students discover that the value of b represents the y-intercept and the value of m represents the slope of the line. Also the concept of x-intercept was developed.

Lesson 4. This lesson was designed to help students discover the behavior of a straight line when the slope is zero, positive or negative.

Lesson 5. Parallel and perpendicular lines. The fact that: (a) parallel lines have equal slopes, and (b) if the slope of a line is m then the slope of a line perpendicular to it is -1/m was introduced in this lesson.

Lesson 6. This lesson focused on concepts related to spatial visualization ability. The concepts of reflection and rotation were discussed.

Lesson 7. This lesson was designed to show the students that the graph of a given linear function is independent of the used scale.

Lesson 8. In this lesson the students studied the concepts of slope, y-intercept and x-intercept algebraically.

Procedures

A week before the treatment began, each student in the experimental group received a graphing calculator. This allowed the students to become familiar with the calculator before the study of graphing. In addition, the investigator met several times with the teachers who were in charge of the two groups. The purposes of these meetings were to explain the objectives of the study, its procedures and to familiarize the teachers with the treatment.

The pretest was followed by the instructional treatment. The teachers completed the eight lessons as described. These lessons were taught during the regular mathematics instruction period and were part of the normal curriculum for those students. The same content was taught to each group. The treatment variable was the method of presentation. The experimental group received its instruction using the graphing calculator. The control group received the traditional method of plotting points to graph a linear function.

The last two days of the study were used for posttesting. The students took a posttest on linear functions, the Card Rotation Test and completed the two questionnaires about attitudes toward mathematics and calculators.

Six homework assignments (Appendix E) were also given to the students during the duration of the study. These assignments covered the material presented in lessons 1, 2, 3, 4, and 5. The assignments consisted of 15 problems each. Students were asked to do as many problems as they could in a one-half hour period.

CHAPTER 4

ANALYSIS OF THE DATA

An analysis of the data collected during the study is included in this chapter. The analysis provides information on the following five aspects:

1. Mathematics achievement in graphing linear functions
2. Use of calculator for homework assignments
3. Spatial visualization ability
4. Attitudes toward mathematics
5. Attitudes toward calculators

Sex differences on four of the aspects were also analyzed. The statistical analysis of the ten hypotheses stated in Chapter 1 will be presented. The hypotheses were tested using the Statistical Analysis System (SAS). For the purposes of the statistical analysis it was included only data of students from whom a complete set of pretest and posttest scores for each of the measurements were obtained.

Testing of Hypotheses

Each hypothesis was tested at the alpha=0.05 level of significance. The hypotheses related to linear functions, spatial visualization, and attitudes were analyzed using General Linear Model procedures. The statistical model to represent the data for testing the hypotheses was the following: $Y=A+B_1X_1+B_2X_2+B_3X_3+B_4B_4+E$ where; Y is the observed score on the posttest. X_1 is the variable representing the pretest score for each individual. X_2 is a dummy variable that represents group membership, $X_2=1$ if the scores are from an individual in the experimental group and $X_2=0$ if the scores are from an individual in the control group. X_3 is a dummy variable representing sex membership, $X_3=1$ if the scores are from a male student, $X_3=0$ if the scores are from a female student. X_4 is the interaction between group membership and sex membership. The variable E represents the error or residual term. Analysis of variance was used to test the hypotheses related to homework assignments.

However, the first analysis done to each set of descriptive statistics was an independent sample t-test to determine equivalency between the groups. It was hypothesized that

1. There is no significant difference between the experimental group and the control group in knowledge about linear functions or in spatial visualization ability prior to the study.

2. There is no significant difference between males and females in knowledge about linear functions or in spatial visualization ability prior to the study.

3. There is no significant difference between the experimental group and the control group in attitudes toward mathematics or toward calculators prior to the study.

4. There is no significant difference between males and females in attitudes toward mathematics or toward calculators prior to the study.

Table 4.1 represents the descriptive statistics for the experimental group and the control group on the pretest and posttest about linear functions.

Table 4.1

Descriptive Statistics of the Pretest
and Posttest about Linear Functions
(Experimental Group vs. Control Group)

Group	N	Maximum Possible Score	Mean	S.D.	Standard Error
Control					
Pre	20	24	9.20	1.20	0.27
Post	20	24	14.70	2.58	0.58
Experimental					
Pre	26	24	4.38	1.33	0.26
Post	26	24	14.07	1.32	0.26

Table 4.2 shows the descriptive statistics for males and females on the pretest and posttest about linear functions. The t-test analysis for testing pretest differences in knowledge about linear functions between the experimental and control groups revealed the following t value, t cal.=-12.62. For 44 degrees of freedom t crit.=-2.021. Therefore, the hypothesis stating that there was no significant difference between the groups in knowledge about linear functions prior to the study is rejected. The t-test analysis for testing pretest differences between males and females in knowledge about linear functions produced the following results: t cal.=0.3848, t crit.=2.201. Therefore, there is no evidence to suggest a significant difference between males and females in knowledge about linear functions prior to the study.

H_1: There is no significant difference in performance between the control group and the experimental group in graphing linear functions and in solving problems related to linear functions.

H_2: There is no significant difference in performance between males and females in graphing linear functions and in solving problems related to linear functions.

Table 4.2

Descriptive Statistics of the Pretest
and Posttest about Linear Functions
(Males vs. Females)

Group	N	Maximum Possible Score	Mean	S.D.	Standard Error
Male					
Pre	18	24	6.67	3.25	0.77
Post	18	24	14.39	2.28	0.54
Female					
Pre	28	24	6.36	2.37	0.45
Post	28	24	14.32	1.79	0.34

GLM procedures for group differences and sex differences in knowledge about linear functions produced the following results (Table 4.3).

Based on the results the group membership, experimental versus control and the sex membership, males versus females were not significant. That means that there is no significant difference between the control group and the experimental group or between males and females in knowledge about linear functions. Therefore, it may be concluded that there is not sufficient evidence to reject either H_1 or H_2.

Table 4.3

General Linear Model Procedures Results
(Linear Functions)

Source	SS (Type III)	Df	MS	F	p
Group	2.65	1	2.65	0.65	0.4343
Sex	0.04	1	0.04	0.01	0.9209
Pre	0.50	1	0.50	0.12	0.7262
Grp*Sex	2.33	1	2.33	0.57	0.4537
Error	166.84	41	4.07		

Table 4.4 contains the descriptive statistics for the experimental and the control group on the pretest and the posttest measuring spatial visualization ability.

Table 4.5 shows the descriptive statistics for males and females on the pretest and posttest measuring spatial visualization ability.

The t value obtained to test pretest differences in spatial visualization ability between the experimental and the control group using this data was 0.2845. The critical t value was 2.021. Hence, the evidence was not good enough to reject the hypothesis stating that there is no significant difference between the groups in spatial visualization ability prior to the study. Similar results were obtained for pretest differences in spatial visualization ability between males and females with a t cal.=0.5545 and a t crit.=2.201.

Table 4.4

Descriptive Statistics of the Pretest and Posttest about Spatial Visualization (Control Group vs. Experimental Group)

Group	N	Maximum Possible Score	Mean	S.D.	Standard Error
Control					
Pre	20	112	47.8	33.12	9.99
Post	20	112	59.0	33.13	10.04
Experimental					
Pre	22	112	49.9	22.22	3.99
Post	22	112	69.1	21.06	3.72

H_3: There is no significant difference in spatial visualization ability between the control group and the experimental group.

H_4: There is no significant difference in spatial visualization ability between males and females.

GLM procedures for group differences and sex differences in spatial visualization ability showed the following results (Table 4.6).

Table 4.5

Descriptive Statistics of the Pretest
and Posttest about Spatial Visualization
(Males vs. Females)

Group	N	Maximum Possible Score	Mean	S.D.	Standard Error
Male					
Pre	15	112	52.13	21.25	5.49
Post	15	112	65.94	16.65	4.30
Female					
Pre	27	112	47.78	27.26	5.25
Post	27	112	66.70	28.61	5.50

Table 4.6

General Linear Model Procedures Results
(Spatial Visualization)

Source	SS (Type III)	Df	MS	F	p
Group	264.81	1	264.81	4.10	0.0489*
Sex	4.24	1	4.24	0.07	0.7992
Pre	23731.46	1	23731.46	367.41	0.0001*
Grp*Sex	219.61	1	219.61	3.40	0.0732
Error	2389.84	37	64.59		

*Significant p<.05

Group membership, experimental group versus control group was significant. Therefore, it may be concluded that there is a significant difference in the developing of spatial visualization ability between the two groups. H_3 states that such a difference does not exist. Hence, H_3 is rejected. Sex membership, males versus females was not significant. Therefore, there is not sufficient evidence to reject H_4. Table 4.7 shows the descriptive statistics for the experimental and control group for attitudes toward mathematics.

Table 4.7

Descriptive Statistics (Attitudes toward Mathematics)
(Experimental Group vs. Control Group)

Group	N	Mean	S.D.	Standard Error
Control				
Pre	20	3.22	0.467	0.10
Post	20	3.21	0.533	0.12
Experimental				
Pre	19	3.24	0.380	0.09
Post	19	3.35	0.395	0.09

Table 4.8 shows the descriptive statistics for males and females for attitudes toward mathematics.

The t-test analysis for testing pretest differences in attitudes toward mathematics between groups produced the following results: t cal.=0.137, t crit.=2.201. Therefore, there is not sufficient evidence to suggest a significant difference between the treatment groups in attitudes toward mathematics prior to the study. Similar results were obtained for pretest differences in attitudes toward mathematics between sex membership with a t cal.=0.672 and a t crit.=2.201.

Table 4.8

Descriptive Statistics (Attitudes toward Mathematics)
(Males vs. Females)

Sex	N	Mean	S.D.	Standard Error
Male				
Pre	16	3.18	0.442	0.11
Post	16	3.19	0.533	0.13
Female				
Pre	23	3.27	0.412	0.09
Post	23	3.33	0.410	0.09

H_5: There is no significant difference between the experimental group and the control group in attitude changes toward mathematics.

H_6: There is no significant difference between males and females in attitude changes toward mathematics.

GLM procedures for group differences and sex differences in attitude changes toward mathematics produced the following results (Table 4.9).

Table 4.9

General Linear Model Procedures Results
(Attitudes toward Mathematics)

Source	SS (Type III)	Df	MS	F	p
Group	0.2399	1	0.2339	1.76	0.1937
Sex	0.115	1	0.0115	0.08	0.7729
Pre	2.9339	1	2.9339	21.5	0.0001*
Grp*Sex	0.0791	1	0.0791	0.58	0.4516
Error	4.64	34	0.1365		

*Significant p<.05

Based on those results the group membership, experimental versus control and the sex membership, males versus females were not significant. That means that there is no significant difference between the control and the experimental groups or between males and females in attitude changes toward mathematics. Therefore, it may be concluded that there is not sufficient evidence to reject either H_5 or H_6.

Table 4.10 shows the descriptive statistics for the control and experimental group for attitudes toward calculators.

Table 4.10

Descriptive Statistics (Attitudes toward Calculators)
(Control Group vs. Experimental Group)

Group	N	Mean	S.D.	Standard Error
Control				
Pre	20	2.90	0.408	0.09
Post	20	2.91	0.490	0.11
Experimental				
Pre	19	2.99	0.296	0.07
Post	19	3.20	0.395	0.09

The t-test analysis for testing pretest differences in attitudes toward calculator between treatment groups revealed the following results: t cal.=0.841, t crit.=2.201. Therefore, there is not sufficient evidence to reject the hypothesis stating that there was no significant differences between the treatment groups in attitudes toward calculator prior to the study. Similar results were obtained for pretest differences in attitudes toward calculator between sex membership with a t cal.= -0.028 and a t crit.=-2.201.

Table 4.11 shows the descriptive statistics for males and females for attitudes toward calculators.

Table 4.11

Descriptive Statistics (Attitudes Toward Calculators)
(Males vs. Females)

Sex	N	Mean	S.D.	Standard Error
Male				
Pre	16	2.95	0.223	0.06
Post	16	3.14	0.407	0.10
Female				
Pre	23	2.95	0.431	0.09
Post	23	2.94	0.497	0.10

H_7: There is no significant difference between the experimental group and the control group in attitude changes toward calculators.

H_8: There is no significant difference between males and females in attitudes changes toward calculators.

GLM procedures for group differences and sex differences in attitude changes toward calculators were as follows (Table 4.12):

Table 4.12

General Linear Model Procedures Results
(Attitudes toward Calculators)

Source	SS (Type III)	Df	MS	F	p
Group	0.7176	1	0.7176	4.40	0.0436*
Sex	0.2548	1	0.2548	1.56	0.2201
Pre	1.7272	1	1.7272	10.58	0.0026*
Grp*Sex	0.0005	1	0.0005	0.00	0.9557
Error	5.5518	34	0.1632		

*Significant $p<.05$

Based on those results group membership, experimental versus control was significant. Hence, it may be concluded that treatment had an effect in attitude changes toward calculators. H_7 states that no significant difference exists between the groups. Therefore, H_7 is rejected.

Results showed that sex membership, males versus females was not significant. Hence, it may be concluded that there is no significant difference in attitude changes toward calculators between males and females. This conclusion does not contradict hypothesis H_8. Therefore, there is not sufficient evidence to reject H_8.

H_9: There is no significant difference in the number of homework problems completed between the control and the experimental groups.

H_{10}: There is no significant difference in the percent of homework problems solved correctly between the control and the experimental groups.

Table 4.13 contains the average number of problems completed by both the experimental group and the control group. In addition, it includes the average number of correct problems and the percent of homework problems solved correctly for each group.

An analysis of these data showed that the average number of problems completed by each group was 11 for the control group and 15 for the experimental group. The average number of problems solved correctly was 8.8 for the control group and 13.3 for the experimental group.

Table 4.13

Average Number of Problems Completed and of Problems Correct and Percent of Problems Solved Correctly in Homework Assignments

Assignment	Problems Completed	Problems Correct	Percent of Problems Solved Correctly
1	C:15 E:15	C:13 E:15	C: 86 E:100
2	C:12 E:15	C:10 E:14	C: 83 E: 93
3	C:11 E:15	C: 8 E:15	C: 73 E:100
4	C:11 E:15	C: 9 E:15	C: 82 E:100
5	C:11 E:15	C: 9 E:11	C: 82 E: 73
6	C: 9 E:15	C: 5 E:15	C: 56 E:100

An analysis of variance performed to these data produced the following results (Table 4.14, Table 4.15).

Table 4.14

Analysis of Variance Results
(Homework Problems Completed)

Source	Df	SS	MS	F
Between Groups	1	163.90	163.90	114.15*
Within Groups	39	56.00	1.44	
Total	40	219.90		

*Significant p<.05

Table 4.15

Analysis of Variance Results
(Percent of Homework Problems Solved Correctly)

Source	Df	SS	MS	F
Between Groups	1	2960.5	2960.5	96.54*
Within Groups	39	1196.0	30.7	
Total	40	4156.5		

*Significant p<.05

The findings revealed that the experimental group performed significantly different in both, the number of homework problems completed and the percent of problems solved correctly. H_9 and H_{10} state that such differences do not exist. Therefore, H_9 and H_{10} are rejected.

Summary

The data collected during the study were analyzed in this chapter. Two statistical analyses were used to analyze the data. General Linear Model procedures were used to test the first eight hypotheses stated in Chapter 1. Analysis of variance was used to analyze the data related to homework assignments.

Results of these analyses revealed the following (a) there was no significant difference between group membership and between sex membership in achievement in graphing linear functions, (b) there was a significant difference between groups in spatial visualization ability, but there was no significant difference between sex membership in spatial visualization ability, (c) there was a significant difference between groups in attitudes toward calculator, (d) there was no significant difference between groups in attitudes toward mathematics, (e) there was no significant difference between sex membership in either attitudes toward calculator or attitude toward mathematics, (f) the experimental group performed significantly better than the control group in homework assignments.

CHAPTER 5

SUMMARY, CONCLUSIONS, RECOMMENDATIONS

The main purpose of this study was to compare the performance of a group of students who received instruction on graphing linear functions using the graphing calculator with a group who received the same instruction but using the traditional method of plotting points. Five areas of comparison were used: mathematics achievement, homework assignments, spatial visualization ability, attitudes toward mathematics, and attitudes toward calculators. A pretest on linear functions written for this study by the investigator, the Card Rotation Test, and two questionnaires about attitudes were given to the students at the beginning of the study. This was followed by the treatment program which consisted of eight lessons about concepts related to graphing linear functions. At the end of the study, a posttest on linear functions was administered. The students were again given the Card Rotation Test and two questionnaires about attitudes. Six homework assignments were also given during the duration of the study.

Two statistical procedures were used to evaluate the data. General Linear Model procedures were used to determine the effect of the treatment program on mathematics achievement, on spatial visualization ability, and on the development of attitude changes toward mathematics and/or calculators. Analysis of variance was used to analyze the data related to homework assignments. A summary of the results obtained in each area and some conclusions follow:

Mathematics Achievement

The question of interest in this area is the following: Is there a difference in students' ability to graph linear functions if the graphing calculator is used as the method of instruction?

Intact classes were used in this study and randomization was not possible. An independent sample t-test analysis was applied to the data to determine if the two groups were significantly different in knowledge about linear functions prior to the beginning of the study. The results reveal that at the .05 level there was a significant difference in pretest means between the two groups (t cal.=-12.62, t crit.=2.021). This implies that the two groups were not equivalent in knowledge on linear functions prior to the study. The group with the higher mean (9.20) on the pretest, the control group, was significantly better than the group with the lower mean

(4.32) on the pretest, the experimental group. For males and females the independent sample t-test shows that males and females were not significantly different in knowledge on linear functions prior to the study.

General Linear Model results reveal that there are no significant differences between the experimental group and the control group or between males and females on the posttest scores. Since the two groups were not equivalent in knowledge about linear functions prior to the study, it is difficult to reach a conclusion about the treatment effect in graphing linear functions. This is due to the fact that some threats to internal validity might be present in the study. In particular, it is possible that statistical regression (tendency for extreme scores to move toward the common mean on subsequent measurement) or the ceiling effect (range of achievement on the test items is limited) have affected the posttest scores. Therefore, it is difficult to determine whether the non significant difference is due to threats to internal validity in the form of preexisting differences or if in fact the treatment had no effect on achievement in linear functions. Additional research in such a way as to avoid those internal validity problems is needed.

Spatial Visualization Ability

Of particular interest in this area is the following: Will use of the graphing calculator help in the development of spatial visualization skills?

Based on the independent sample t-test results it may be concluded that

1. The experimental group and the control group were significantly equivalent in spatial visualization ability prior to the study.

2. Males and females were equivalent in spatial visualization ability prior to the study.

General Linear Model procedures show that there is a significant difference on the posttest scores due to the type of instruction received. Since there are only two groups participating in the study, the group with the higher mean score on the posttest performs significantly better than the group with the lower posttest mean. The experimental group has a posttest mean of 69.06, the control group has a posttest mean of 59.00. Therefore, it may be concluded that although the two groups were equivalent in spatial visualization ability at the beginning of the study, the group using the graphing calculator as the method of instruction developed more spatial skills than the group using the traditional method. Therefore, it may be concluded that, it appears that the graphing calculator is an effective tool in the developing of spatial visualization ability.

Researchers are constantly debating whether males have a better spatial visualization ability than females. The data in this study support that there is no significant difference on the performance of males and females on the Card Rotation Test. This test was used to measure spatial visualization ability. Therefore, it may be concluded that in this sample males and females did not perform differently on spatial visualization skills.

Attitudes

The main concern in this area is whether the use of the graphing calculator to graph linear functions produces changes in attitudes toward mathematics and/or toward the use of calculators.

Statistical results in Chapter 4 reveal that there is a significant difference in posttest means between the experimental and the control groups on attitudes changes toward the use of calculators. The experimental group has a posttest mean of 3.20. The posttest mean for the control group is 2.91. Therefore, the conclusion is that the attitude toward the use of calculators in mathematics instruction is better for the group using the calculator as the method of instruction than for the group using the traditional method. Therefore, the graphing calculator proved to be an effective tool in producing changes in attitudes toward calculator use in the mathematics classroom. The effects which were tested in this area and found to be non-significantly different include the effects of the treatment on developing attitudes toward mathematics both in group membership and in sex membership and the effects of the treatment on producing changes in attitudes toward calculators in males and females.

Homework Assignments and Other Conclusions

An analysis of the data in Table 4.13 shows that the average number of problems completed by each group is 11 for the control group, 15 for the experimental group. The average number of problems solved correctly is as follows: 8.8 for the control group, 13.3 for the experimental group. Analysis of variance procedures were applied to the number of problems completed and to the percent of problems solved correctly. The results show that the experimental group performed significantly better than the control group in both the number of problems completed and the percent of problems solved correctly. Recalling that students were asked to complete the assignment in a one half-hour period, it may be concluded that the graphing calculator appears to be an effective time-saving piece of technology to solve problems related to linear functions.

Another conclusion of the study is that some concepts related to linear functions appeared to be more difficult to the students than others (Appendix F). For example, it was observed that students from both groups had difficulty with items about the relationship between parallelism and slope of a line and the relationship between perpendicularity and slope of a line. Another difficulty encountered among students in both groups was on items related to the identification of different scales for the same linear function. Those difficulties were observed to be consistent throughout the entire study.

Some observations and recommendations should be made based on the results of this study. First of all, generalizations of the findings of this study should be made with caution. This is because of that some threats of internal and external validity due to the limitations stated in Chapter 1 could not be avoided. Second, since the graphing calculator is a new piece of technology other studies are needed in order to determine its place in the mathematics curriculum.

Further studies should focus on other concepts to insure that the differences found in this study were due to the type of instruction received by the students and not simply on the concepts that were taught. Those studies should include larger samples and different groups of students. In addition, there is a need for longer term studies.

Finally, the findings of this research provide evidence that indicates that the graphing calculator could be as effective to teach skills in graphing linear functions as the traditional method of plotting point. Although the pre and post measures showed no significant difference in graphing linear functions, the graphing calculator was more effective than the traditional method in developing spatial visualization ability and attitudes. In addition, it saved time and helped the students to do homework assignments more efficiently.

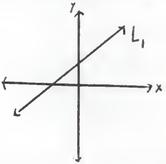
APPENDIX A

PRETEST

NUMBER ____________________ SEX _______

Instructions: Circle the letter which best answers each of the following problems:

1. The slope (steepness) of the line L_1 in the diagram at the right is:



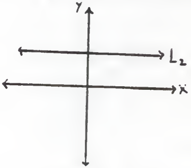
A) a positive number

B) a negative number

C) zero

D) none of the above

2. The slope of the line L_2 in the diagram at the right is:



A) a positive number

B) a negative number

C) zero

D) none of the above

3. The slope of the line represented by the equation $y=3x+2$ is:

A) 2/3

B) 3/2

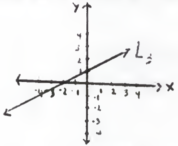
C) 2

D) 3

4. The slope of the line represented by the equation 6x+4y=8 is:
   - A) 2
   - B) 4
   - C) -3/2
   - D) 3/2

5. The slope of the line passing through the points (3,2) and (2,3) is:
   - A) -1
   - B) 1
   - C) 2
   - D) 3

6. The y-intercept of the line L_3 (above) is:
   - A) -2
   - B) -1
   - C) 0
   - D) 1



7. The x-intercept of the line L_3 (above) is:
   - A) -2
   - B) -1
   - C) 0
   - D) 1

8. The y-intercept of the line represented by the equation y=3x+5 is:

   A) 3

   B) 3/5

   C) 5/3

   D) 5

9. The y-intercept of the line represented by the equation 2x+3y=6 is:

   A) -2/3

   B) 2/3

   C) 2

   D) 3

10. The x-intercept of the line represented by the equation 2x+3y=6 is:

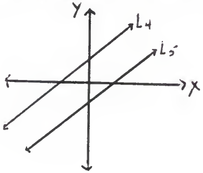
   A) -2/3

   B) 2/3

   C) 2

   D) 3

Problems 11 and 12 refer to the following figure:



11. The lines L_4 and L_5 are:

    A) perpendicular

    B) parallel

    C) neither perpendicular nor parallel

    D) vertical

12. If the slope of the line L_4 is 3, then the slope of the line L_5 is:

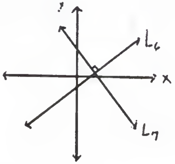
    A) -1/3

    B) 1/3

    C) 3

    D) 6

Problems 13 and 14 refer to the following figure:



13. The lines L_6 and L_7 are:

    A) perpendicular

    B) parallel

    C) neither perpendicular nor parallel

    D) horizontal

14. If the slope of the line L_6 is 2, then the slope of the line L_7 is:

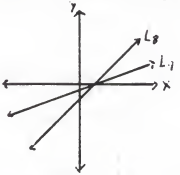
A) -1/2

B) 1/2

C) 2

D) 4

15. Refer to the following figure:



The slope of the line L_8 is:

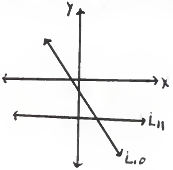
A) greater than the slope of the line L_9

B) less than the slope of the line L_9

C) equal to the slope of the line L_9

D) -3

16. Refer to the following figure:



The **slope** of the line L_{10} is:

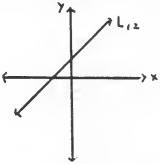
A) greater than the slope of the line L_{11}

B) less than the slope of the line L_{11}

C) equal to the slope of the line L_{11}

D) 1

17. Refer to the following figure:



The line L_{12} lies in quadrants:

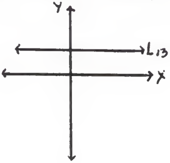
A) two and three only

B) one and two only

C) one and three only

D) one, two and three only

18. Refer to the following figure:



The line L_{13} lies in quadrants:

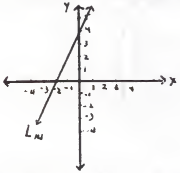
A) one and two only

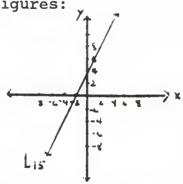
B) one and three only

C) two and three only

D) one, two and three only

19. Refer to the following figures:





The equations representing the lines L_{14} and L_{15} are the same:

A) true

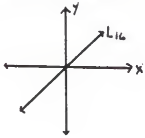
B) false

20. The lines represented by the equations $y=x+1$ and $-2x+2y=2$ are the same.

A) true

B) false

Problems 21 and 22 refer to the following figure:



21. A rotation of 90° (degrees) would move L_{16} to quadrants:

    A) one and two

    B) two and three

    C) two and four

    D) one and four

22. A rotation of 180° (degrees) would move L_{16} to quadrants:

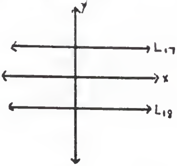
    A) one and two

    B) two and three

    C) two and four

    D) one and three

23. Refer to the following figure:



The line L_{18} is a reflection of the line L_{17} about:

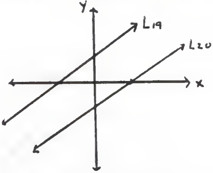
A) the x-axis

B) the y-axis

C) the origin

D) none of the above

24. Refer to the following figure:



The line L_{20} can be obtained from the line L_{19} by a reflection about:

A) the line y=-x

B) the x-axis

C) the y-axis

D) the line y=x

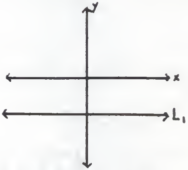
APPENDIX B

POSTTEST

NUMBER ______________________ SEX _______

Instructions: Circle the letter which best answers each of the following problems:

1. The slope (steepness) of the line L_1 in the diagram at the right is:



A) a positive number

B) a negative number

C) zero

D) none of the above

2. The slope of the line represented by the equation y=4x+1 is:

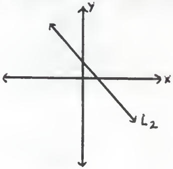
A) 1

B) 1/4

C) 4

D) 0

3. The slope of the line L_2 in the diagram at the right is:



A) a positive number

B) a negative number

C) zero

D) none of the above

4. The slope of the line represented by the equation $2x+3y=6$ is:

A) 2

B) 3

C) -2/3

D) -3/2

5. The slope of the line passing through the points (5,3) and (4,8) is:

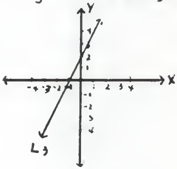
A) 3

B) -5

C) 5

D) 8

6. The x-intercept of the line L_3 in the diagram at the right is:



A) -2

B) -1

C) 0

D) 2

7. The y-intercept of the line L_3 (above) is:

A) -2

B) -1

C) 0

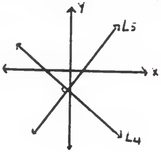
D) 2

8. The y-intercept of the line represented by the equation y=2x+7 is:
    - A) 2
    - B) 2/7
    - C) 7/2
    - D) 7

9. The x-intercept of the line represented by the equation 6x+4y=8 is:
    - A) -2/3
    - B) -2
    - C) 2
    - D) 4/3

10. The y-intercept of the line represented by 6x+4y=8 is:
    - A) -2/3
    - B) 2/3
    - C) 2
    - D) 4/3

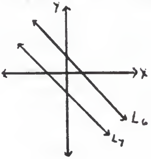
Problems 11 and 12 refer to the following figure:



11. The lines L_4 and L_5 are:
    - A) parallel
    - B) perpendicular
    - C) neither parallel nor perpendicular
    - D) horizontal

12. If the slope of L_4 is 4, then the slope of L_5 is:
    - A) -1/4
    - B) 1/4
    - C) 4
    - D) -4

Problems 13 and 14 refer to the following figure:



13. The lines L_6 and L_7 are:
    - A) parallel
    - B) perpendicular
    - C) neither parallel nor perpendicular
    - D) vertical

14. If the slope of L_6 is 2, then the slope of L_7 is:

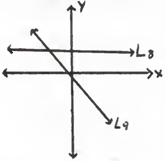
A) -1/2

B) 1/2

C) -2

D) 2

15. Refer to the following figure:



The slope of L_8 is:

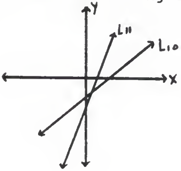
A) greater than the slope of L_9

B) less than the slope of L_9

C) equal to the slope of L_9

D) zero

16. Refer to the following figure:



The slope of L_{10} is:

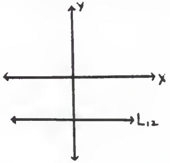
A) greater than the slope of L_{11}

B) less than the slope of L_{11}

C) equal to the slope of L_{11}

D) one

17. Refer to the following figure:



The line L_{12} lies in quadrants:

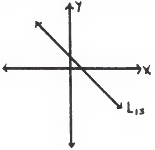
A) two and three only

B) one and three only

C) three and four only

D) two, three and four only

18. Refer to the following figure:



The line L_{13} lies in quadrants:

A) two and four only

B) one and four only

C) one and two only

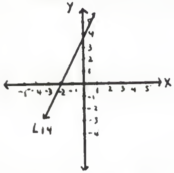
D) one, two and four only

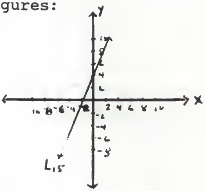
19. The lines represented by the equations $y=2x-3$ and $4x+2y=6$ are the same:

A) true

B) false

20. Refer to the following figures:



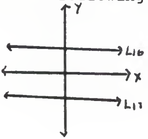


The equations representing the lines L_{14} and L_{15} are the same:

A) true

B) false

21. Refer to the following figure:



The line L_{16} is a reflection of the line L_{17} about:

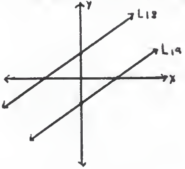
A) the line y=x

B) the x-axis

C) the y-axis

D) the line y=-x

22. Refer to the following figure:



The line L_{19} can be obtained from the line L_{18} by a reflection about:

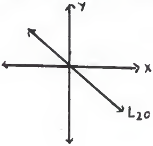
A) the line y=x

B) the y-axis

C) the line y=-x

D) the x-axis

Problems 23 and 24 refer to the following figure:



23. A rotation of 90° (degrees) would move L_{20} to quadrants:

    A) two and four

    B) one and two

    C) one and three

    D) two and three

24. A rotation of 180° (degrees) would move L_{20} to quadrants:

    A) two and four

    B) one and two

    C) one and three

    D) two and three

APPENDIX C
ATTITUDES QUESTIONNAIRE

ATTITUDES TOWARD MATHEMATICS

Please do not write your name in this questionnaire as you will not be identified by name. Therefore, feel free to be very honest.

Express, on a five point scale, the extent of your agreement between the feeling expressed in each of the following statements on your own personal feelings.

1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree, 5 = strongly agree, 9 = no response

1. I think mathematics is fun.
2. I usually understand what we are talking about in mathematics classes.
3. I really want to do well in mathematics.
4. Mathematics helps one to think logically.
5. It scares me to have to take mathematics.
6. Working with numbers makes me happy.
7. Learning mathematics involves mostly memorizing.
8. I feel good when I solve a mathematics problem by myself.
9. I like to help others with mathematics problems.
10. I am not so good at mathematics.

ATTITUDES TOWARD CALCULATORS

Please do not write your name in this questionnaire as you will not be identified by name. Therefore, feel free to be very honest.

Express, on a five point scale, the extent of your agreement between the feeling expressed in each of the following statements on your own personal feelings.

1 = strongly disagree, 2 = disagree, 3 = undecided,
4 = agree, 5 = strongly agree, 9 = no response

1. Everyone should learn something about calculators.
2. Solving problems is more fun if you use a calculator.
3. If you use a calculator you do not have to learn to compute.
4. Using calculators makes learning mathematics more mechanical and boring.
5. It is less fun to learn mathematical ideas if you use a calculator.
6. Using a calculator can help you learn many different mathematical topics.
7. Calculators solve problems better than people do.
8. I dislike calculators because of my experience with them.
9. People are becoming too dependent on calculators.
10. Learning about how to use calculators will help me to prepare for the future.

ANSWER SHEET

NUMBER ____________________ SEX ________

1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree, 5 = strongly agree, 9 = no response

Attitudes toward mathematics

1) 1 2 3 4 5 9
2) 1 2 3 4 5 9
3) 1 2 3 4 5 9
4) 1 2 3 4 5 9
5) 1 2 3 4 5 9
6) 1 2 3 4 5 9
7) 1 2 3 4 5 9
8) 1 2 3 4 5 9
9) 1 2 3 4 5 9
10) 1 2 3 4 5 9

Attitudes toward calculators

1) 1 2 3 4 5 9
2) 1 2 3 4 5 9
3) 1 2 3 4 5 9
4) 1 2 3 4 5 9
5) 1 2 3 4 5 9
6) 1 2 3 4 5 9
7) 1 2 3 4 5 9
8) 1 2 3 4 5 9
9) 1 2 3 4 5 9
10) 1 2 3 4 5 9

APPENDIX D
INSTRUCTIONAL LESSONS

LESSON #1: HORIZONTAL LINES

The activities in this lesson deal with graphing equations for horizontal lines. The students will work with equations in the form y=b, where b is a constant. They will discover that equations of the form y=b are parallel to the x-axis with y-intercept at b.

CLEAR THE SCREEN AND SELECT AN APPROPRIATE SCALE.

For example: x-min:-8, x-max:8, scale:1, y-min:-G, y-max:8, scale:1

1. Graph the following equations:

   $y = 4$

   $y = -3$

   $y = 1$

   $y = 1/2$

   $y = -.5$

Question: Predict where the graphs of the following equations would fall in the Cartesian plane:

"CLEAR THE SCREEN"

$y = 2.5$

$y = -8$

$y = 8$

$y = 2$

Graph the given equation. Were your predictions close?

Question: There is an important relationship between a line in the form y=b and the x-axis. What is this relationship?

Notice that all lines parallel to the y-axis are parallel to each other.

There is an interesting relationship between each equation given above and the point where its graph crosses the y-axis.

Question: What are the coordinates of the point where each graph crosses the y-axis?

Definition: The value of b is called the y-intercept and (0,b) are called the coordinates of the y-intercept.

Name the y-intercept for each equation on the screen.

Ask students to suggest more equations of lines parallel to the x-axis. Graph the equations whose lines will fall within the limit of the screen. Identify the coordinates for the y-intercept and state the y-intercept for each question.

Question: What is the equation for the x-axis?

LESSON #2: LINES THROUGH THE ORIGIN

This activity introduces linear equations in the form y=mx+b, where b=0.

A) Clear the screen

B) Select an appropriate scale. (e.g. x-min=-10, x-max=10, scale=1, y-min=-10, y-max=10, scale=1)

C) Graph the following equations:

   1) y = .25x
   2) y = .5x
   3) y = x
   4) y = 3x
   5) y = 7x

D) Question: What happens to the steepness of the lines as the coefficient of the variable x increases?

E) Definition: This "steepness" is called the slope of the line. It is represented in the equation y=mx by the coefficient of x.

F) Clear the screen

G) Predict the graph of the following equations before entering into the calculator. Graph them, were your predictions close? State the slope for each equation.

1) $y = x$
2) $y = 5x$
3) $y = .1x$
4) $y = 1/3x$
5) $y = 6x$

H) Equations of the form $y=-mx$ are now introduced.

I) Clear the screen.

J) Graph the following equations:

1) $y = -.25x$
2) $y = -.5x$
3) $y = -x$
4) $y = -3x$
5) $y = -6x$

K) Question: What is happening to the slope of these equations?

L) Clear the screen.

M) Predict where the graphs of $y=-5x$ and $y=.9x$ would fall in the Cartesian plane. Graph the two equations. Were you close?

N) Follow the same procedure with the equations $y=5x$ and $y=-.9x$

O) Clear the screen.

P) **Compare the** graphs of: (Remember to clear the screen after each comparison)

1) $y = x$ and $y = x$
2) $y = -4x$ and $y = 4x$
3) $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$

Q) Question: What can be said in general about the graphs of $y=mx$ and $y=-mx$? For lines of the form $y=mx$ (i.e., y-intercept=0), if the slope is positive in what quadrants will the graph lie? What about if the slope is negative?

LESSON #3: LINEAR EQUATIONS OF THE FORM y=mx+b

These activities are designed to help students discover;

1) that the value of b in y=mx+b is the y-intercept of the graph of the equation.
2) the coordinates for the x-intercept of the graph of the equation.

A) Question: All the lines we graphed in lesson 2 had one point in common. What was that point? Do the graphs of all linear equations pass through the origin?

B) Clear the screen and using the same scale for lesson 2, graph the following equations.

1) $y = .2x+3$
2) $y = (\frac{1}{2})x+3$
3) $y = x+3$
4) $y = 3x+3$
5) $y = 5x+3$

C) Question: What point do these lines have in common?

D) Clear the screen.

E) Repeat the procedure with the equations:

1) $y = (-1/10)x-2$
2) $y = -.5x-2$
3) $y = -x-2$
4) $y = -2x-2$
5) $y = -4x-2$

F) Clear the screen.

G) Recall that we call the y-coordinate of the point where the graph of an equation crosses the y-axis the y-intercept. The coordinates of the y-intercept are $(0,b)$.

H) Question: State the coordinates of the y-intercept and predict where the graph of each of the following equations would fall in the Cartesian plane. Verify graphing the equation.

1) $y = x + 5$
2) $y = -2x - \frac{1}{2}$
3) $y = 3x + 1$
4) $y = 2x$

I) Question: When a linear equation is in the form $y=mx+b$, what are the coordinates of the y-intercept?

J) Notice that the slope m has no effect on the y-intercept.

K) Clear the screen.

L) Graph the equations

1) $y = (-\frac{1}{2})x + 2$
2) $y = x - 4$
3) $y = (3/2)x - 6$

M) Question: What point do these graphs have in common? What are the coordinates of the point where each graph crosses the x-axis?

N) **Definition:** The value of x where the line crosses the **x-axis is** called the x-intercept and (x,0) are the coordinates of the x-intercept.

LESSON #4: POSITIVE OR NEGATIVE SLOPE

This lesson is designed to help students discover that:

1) lines with positive slopes go up as x increases
2) lines with negative slopes go down as x increases
3) lines with slope=0 are horizontal

A) Clear the screen.

B) Select the same scale as in previous lessons.

C) Review the concepts of slope and y-intercept using the equation $y=5x+3$

D) Clear the screen.

E) Graph the following equations:

1) $y = 3x - 1$
2) $y = x + 2$
3) $y = (1/3)x - 5$
4) $-4x + y = 5$

Observation: For the last equation, recall that the calculator input is of the form $y=mx+b$. Therefore show them how to change $-4x+y=5$ to $y=4x+5$.

F) Questions: What do these graphs have in common? Where does each line fall in the Cartesian plane?

G) Notice that:

1) when the slope is positive, the line goes up as x increases

   2) for positive slopes, if the y-intercept is positive the line lies in quadrants 1, 2, and 3; if the y-intercept is negative the line lies in quadrants 1, 3, and 4.

H) Clear the screen.

I) Graph the equations:

   1) $y = -2x - 3$
   2) $y = -x + 1$
   3) $y = -4x + 2$
   4) $2x + 3y = 1 \Longrightarrow y = (-2/3)x + 1/3$

J) Questions: What do these lines have in common? Where does each line fall in the Cartesian plane?

K) Notice that:

   1) when the slope is negative, the line goes down as x increases.
   2) for negative slopes, if the y-intercept is positive the line lies in quadrants 1, 2, and 4; if the y-intercept is negative the line lies in quadrants 2, 3, and 4.

L) Clear the screen.

M) Graph the equations:

   1) $y = 2$
   2) $y = -4$
   3) $y = -3$
   4) $y = 5$

N) Repeat the same questions as before.

O) Notice that:

   1) horizontal lines have slope = 0

   2) the line lies in quadrants 1 and 2 if b is positive; the line lies in quadrants 3 and 4 if b is negative.

LESSON #5: PARALLEL AND PERPENDICULAR LINES

The following activities will help students to discover that:

1) parallel lines have equal slopes.
2) if the slope of a line is m, then a line perpendicular to it has slope -1/m.

A) Clear the screen.

B) Select the same scale as before.

C) Predict the location of the following lines in the Cartesian plane. Then graph the equations.

1) $y = 2x + 1$
2) $y = 2x - 8$
3) $y = 2x - 1$
4) $y = 2x$

D) Questions: What do you notice about the graphs of all of these equations? What do the equations of these lines have in common? Can you make a statement to summarize these findings? For which of the following equations will the graphs be parallel lines?

1) $y = 4x - 4$
2) $y = -4x - 4$
3) $y = 4x + 3$
4) $y = 8x + 2$

E) Verify by graphing the equations.

F) Clear the screen.

G) Graph the equations:

1) $y = 2x - 1$

2) $y = (-\frac{1}{2})x + 3$

Observation: Graphs of equations of perpendicular lines will not always perpendicular because of the distortion of the screen.

H) Clear the screen.

I) Graph

1) $y = (-3/4)x + 4$

2) $y = (4/3)x - 2$

J) Questions: How are the graphs of each of the above sets of equations related? How are the slopes in each of the above sets of equations related? Can you make a statement to generalize these findings? Which of the following linear equations will produce perpendicular lines? Graph to test your conjecture.

1) $y = 3x + 4$

2) $y = (1/3)x - 4$

3) $y = -3x - 1/4$

4) $6y = -2x + 12$

LESSON #6: ROTATIONS AND REFLECTIONS

These activities will help the student to determine:

1) where a line will lie after we rotate the plane about the origin.
2) what is the line of reflection of a given line.

A) Clear the screen.

B) Use the same scale as before.

C) Graph the equations:

1) $y = 3x$
2) $y = -3x$

D) Questions: In what quadrants the graph of $y=3x$ lies? How about the graph of $y=-3x$?

E) Notice that the line $y=-3x$ can be obtained from the line $y=3x$ by rotating the plane about the origin certain number of degrees.

F) Clear the screen and repeat the procedure with the equations:

1) $y = 5x$
2) $y = -5x$

G) Definition: A rotation is the act or process of turning around a center (the origin in our case).

H) Clear the screen.

I) Graph the equations:

1) $y = 2$
2) $y = -2$

J) **Question:** What relationship do you observe between the two graphs?

K) Notice that one line is a reflection of the other about the line $y=0$ (i.e., the x-axis).

L) Clear the screen.

M) Graph the equations:

   1) $y = (\frac{1}{2})x + 2$
   2) $y = (\frac{1}{2})x - 2$
   3) $y = (\frac{1}{2})x$

N) Question: What can you say about the graphs of $y=(\frac{1}{2})x+2$ and $y=(\frac{1}{2})x-2$?

O) Notice that one line is a reflection of the other about the line $y=(\frac{1}{2})x$.

P) Definition: A reflection is the appearance of a figure after it has been flipped about some line. This line is called the line of reflection and it acts like a mirror.

R) Clear the screen and repeat the procedures with the following equations if time permits.

   1) $y = (1/3)x + 5$
   2) $y = (1/3)x - 5$
   3) $y = (1/3)x$

LESSON #7: USING DIFFERENT SCALES

A) Clear the screen.

B) Use the same scale as before.

C) Graph the equation y=(-2/3)x+2

D) Questions: What is the slope of this line? What is the y-intercept? What is the x-intercept?

E) Change the scale. Use x-min=-10, x-max=10, scale=2, y-min=-10, y-max=10, scale=2.

F) Graph the same equation again.

G) Repeat the questions given above.

H) Make the observation that now each dot represents 2 units.

I) Change the scale again using the same minimum and maximum values for x and y but with scale=4.

J) Graph the equation y=(-2/3)x+2 again.

K) Ask the questions described above.

L) Guide the students to realize that the graph of y=(-2/3)x+2 is the same.

M) Repeat the procedures with other equations if time permits.

LESSON #8: SLOPE, Y-INTERCEPT, AND X-INTERCEPT USING FORMULAS

The activities in this lesson are exercises in finding the slope (m), the y-intercept and the x-intercept using algebraic procedures.

I. Slope given two points (x_1, y_1) & (x_2, y_2) on the line using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_2 \neq x_1$$

A) Recall that the slope of a line whose equation is given in the form y=mx+b is m. Now we will find out how to compute the slope when two points through the line are known.

B) Show the use of the formula computing the slope for the line through the points (2,3) and (4,5).

C) Graph the line y=x+1 (i.e., the line through the given points)

D) Notice that we have referred to the slope as the steepness of the graph. We can measure slope or steepness by comparing how far the graph rises along the y-axis for each unit it travels along the x-axis. This is usually referred to as rise/run.

E) Guide the students to realize that in this case for each change of 1 unit in the x-axis, the change in the y-axis is also 1.

F) **Repeat the** procedure with the line through the points (2,4) and (3,7) (i.e., y=3x-2)

In this case for each change of 1 unit in the x-axis the change in the y-axis is of 3 units.

G) Recall that the coordinates of the y-intercept are (0,b) and the coordinates for the x-intercept are (x,0). Therefore, to find the y-intercept algebraically we let x=0 and solve for y. Similarly to find the x-intercept, we let y=0 and solve for x.

H) Illustrate the procedures with the equations 2x+3y=6 and x-y=2.

I) Graph the equations to verify.

APPENDIX E
HOMEWORK ASSIGNMENTS

NAME: ______________________________ SEX: _______

HOMEWORK: GRAPH THE FOLLOWING FUNCTIONS. STATE THE COORDINATES FOR THE Y-INTERCEPT.

1) $y = 1$
2) $y = -4$
3) $y = -1/3$
4) $y = 7$
5) $y = 0.4$
6) $y = 6.5$
7) $y = 2/3$
8) $y = 0$
9) $y = -4.2$
10) $y = 3$
11) $y = 3/5$
12) $y = 4$
13) $y = -1$
14) $y = 3.2$
15) $y = -7$

NAME: ______________________________ SEX: ________

HOMEWORK: GRAPH THE FOLLOWING FUNCTIONS. STATE THE COORDINATES FOR THE Y-INTERCEPT AND THE SLOPE FOR EACH FUNCTION.

1) $y= 3x + 2$
2) $y= 2x + 1$
3) $y = x - 5$
4) $y= -2x - 3$
5) $y= 0.5x - 1$
6) $y = (-2/3)x + 2$
7) $y= -0.5x + 6$
8) $y= 0$
9) $y = -4.2$
10) $y= (3/4)x - 4$
11) $y= (\frac{1}{2})x + 1/3$
12) $y = -0.2x -.1$
13) $y= -x - 3$
14) $y= 3.2x + 2.5$
15) $y = -7x + 1/4$

NAME: ______________________________ SEX: _______

HOMEWORK: GRAPH THE FOLLOWING FUNCTIONS.

WHICH ONES HAVE POSITIVE SLOPES?

WHICH ONES HAVE NEGATIVE SLOPES?

WHICH ONES HAVE ZERO SLOPE?

1) $y = 3x + 2$
2) $y = 2x + 1$
3) $y = x - 5$
4) $y = -2x - 3$
5) $y = 0.5x - 1$
6) $y = (-2/3)x + 2$
7) $y = -0.5x + 6$
8) $y = 5$
9) $y = -4.2x - 5$
10) $y = x$
11) $y = (\frac{1}{2})x + 1/3$
12) $y = -5x$
13) $y = -1/2$
14) $y = 3.2x + .25$
15) $y = -7x + 1/4$

NAME: ______________________________ SEX: _______

HOMEWORK: GRAPH THE FOLLOWING FUNCTIONS. STATE THE COORDINATES OF THE X-INTERCEPT AND THE Y-INTERCEPT AND THE SLOPE.

1) $y = 3x + 2$
2) $y = -2x$
3) $y = x - 9$
4) $y = -2x - 3$
5) $y = 0.5x - 4$
6) $y = (-3/2)x + 2$
7) $y = -0.5x - 3$
8) $y = 5$
9) $y = -5.2x - .4$
10) $y = x$
11) $y = (\frac{1}{2})x + 1/3$
12) $y = -3x$
13) $y = -\frac{1}{2}$
14) $y = 3.2x + .25$
15) $y = -7x + 1/4$

NAME: ______________________________ SEX: _______

HOMEWORK: GRAPH THE FOLLOWING FUNCTIONS.

WHAT LINES ARE PARALLEL TO EACH OTHER?

WHAT LINES ARE PERPENDICULAR TO EACH OTHER?

1) $y = x + 3$
2) $y = -2x - 1$
3) $y = x - 5$
4) $y = -x + 4$
5) $y = -2x + 5$
6) $y = \frac{1}{2}x - 5$
7) $y = -0.5x - 3$
8) $y = 5x$
9) $y = 1/3x - 4$
10) $y = (-1/5)x$
11) $y = (\frac{1}{2})x + 1/3$
12) $y = -3x$
13) $y = -\frac{1}{2}$
14) $y = 2/3x - 4$
15) $y = (-3/2)x + \frac{1}{4}$

NAME: ______________________________ SEX: _______

HOMEWORK: FOR THE FOLLOWING EQUATIONS, STATE:

a) the slope

b) the y-intercept

c) the x-intercept

d) those lines that are parallel

e) those lines that are perpendicular

f) those lines which go up as x increases

g) those lines which go down as x increases

h) those lines which remain constant as x increases

1) $y= 3x$

2) $y = 2$

3) $y = -3x + 5$

4) $y= 2x -3$

5) $y = x$

6) $y = -1$

7) $y= (-1/3)x + 4)$

8) $y = 2x - 1$

9) $y = -x$

10) $y= (\frac{1}{2}) + 3$

11) $y = -5x + 1/5$

12) $y = 7x - 8$

13) $y= (1/5) + 5$

14) $y = 4$

15) $y = -5x$

APPENDIX F

p VALUES FOR ITEMS (LINEAR FUNCTIONS)

Item	p Value
1	.67
2	.53
3	.30
4	.30
5	.22
6	.34
7	.43
8	.15
9	.26
10	.24
11	.54
12	.04
13	.30
14	.07
15	.37
16	.39
17	.22
18	.28
19	.08
20	.10
21	.17
22	.13
23	.20
24	.20

p is the proportion of all attempting the item who get the right answer.

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BIOGRAPHICAL SKETCH

Juan L. Vazquez was born on August 29, 1949, in Ciales, Puerto Rico. Here he completed the elementary, middle, and high school studies. He then attended the University of Puerto Rico in Rio Piedras. There he earned the Bachelor of Education degree in secondary mathematics education.

Juan taught mathematics for three years in his home town and then went to the University of Illinois in Urbana-Champaign, where he completed a Master of Education degree in mathematics education. Juan went back to Puerto Rico and after two years as a mathematics coordinator and as a curriculum specialist he took over a position at the University of Puerto Rico in Arecibo, where he taught mathematics for nine years.

Juan then moved to Gainesville, Florida, to begin studies toward the doctorate in mathematics education. While at the University of Florida, he was a teaching assistant for both the Department of Mathematics and the Department of Instruction and Curriculum.

In the fall of 1989, Juan was hired as an Assistant Professor of Mathematics at Barry University in Miami, where he is currently teaching.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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